7. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point A on a smooth horizontal table. The particle P is at rest at the point B on the table, where $AB = l$.

At time $t = 0$, P is projected along the table with speed U in the direction AB.

At time t

- the elastic string has not gone slack
- $BP = x$
- the speed of P is v

(a) Show that

$$v^2 = U^2 - \frac{2gx^2}{l}$$

(4)

(b) By differentiating this equation with respect to x, prove that, before the elastic string goes slack, P moves with simple harmonic motion with period $\pi\sqrt{\frac{2l}{g}}$

(5)

Given that $U = \sqrt{\frac{gl}{2}}$

(c) find, in terms of l and g, the exact total time, from the instant it is projected from B, that it takes P to travel a total distance of $\frac{3}{4}l$ along the table.

(6)

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TOTAL FOR PAPER IS 75 MARKS

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