6: (a) Show, using algebraic integration, that the centre of mass of a uniform **solid** hemisphere H of radius a is a distance $\frac{3}{8}a$ from O, the centre of its plane face.

[*You may assume that the volume of the hemisphere is* $\frac{2}{3}\pi a^3$]

(5)

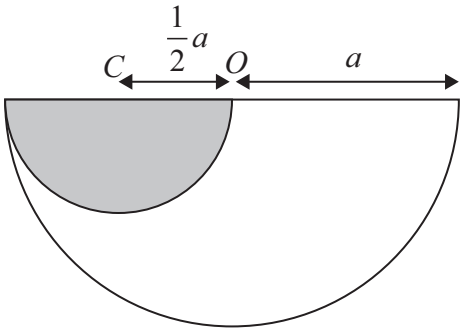


Figure 5

A uniform **solid** S is formed by removing a smaller **solid** hemisphere of radius $\frac{1}{2}a$ from H such that

- the plane face of the smaller hemisphere has centre C and is part of the plane face of H
- $OC = \frac{1}{2}a$

Figure 5 shows a cross section of S, where S is the **unshaded** part.

(b) Show that the centre of mass of S is $\frac{45}{112}a$ from the line through O and C.

(4)

The solid S rests in equilibrium with its curved surface in contact with a rough horizontal plane, as shown in Figure 6.

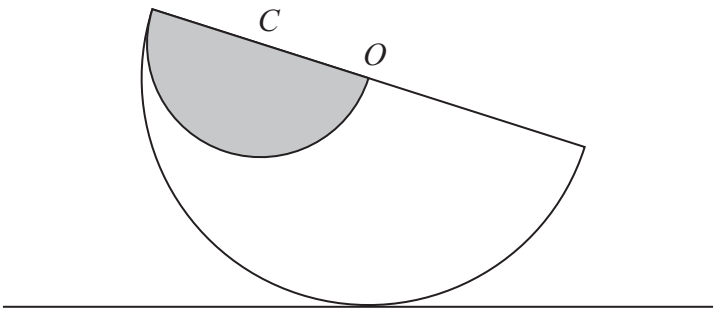


Figure 6

The angle between CO and the horizontal is θ.

(c) Find the exact value of $\tan\theta$.

(5)

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Question 6 continued

(Total for Question 6 is 14 marks)