Question Number	Scheme		Marks
5(a)	$\text{Volume} = \pi\int_0^{\frac{\pi}{2}} y^2\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} \sin^2 x\,\mathrm{d}x$		M1
	$= \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2x)\,\mathrm{d}x$		M1
	$= \pi\left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}} = \pi\left[\left(\frac{\pi}{4} - \frac{1}{4}\sin\pi\right) - \left(0 - \frac{1}{4}\sin 0\right)\right]$		DM1
	$= \frac{\pi^2}{4}$ *		A1*
			(4)
(b)	$\pi\int_0^{\frac{\pi}{2}} y^2 x\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} x\sin^2 x\,\mathrm{d}x$		M1
	Method 1: Double angle formula then integration by parts	$\pi\int_0^{\frac{\pi}{2}} \frac{x}{2}\mathrm{d}x - \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}x\cos 2x\,\mathrm{d}x$	
		$= \left(\pi\left[\frac{x^2}{4}\right]_0^{\frac{\pi}{2}}\right) - \pi\left[\frac{x}{2}\times\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} + \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}\times\frac{1}{2}\sin 2x\,\mathrm{d}x$	M1
		$= \frac{\pi^3}{16}\left(-\pi[0-0] - \pi\left[\frac{1}{8}\cos 2x\right]_0^{\frac{\pi}{2}}\right)$	B1
		$= \left(\frac{\pi^3}{16}\right) - \frac{\pi}{8}[\cos\pi - \cos 0]$	DM1
	Method 2: Integration by parts using result from part (a)	$= \pi\left[\frac{x}{2}\left(x - \frac{1}{2}\sin 2x\right)\right]_0^{\frac{\pi}{2}} - \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\mathrm{d}x$	M1
		$= \frac{\pi^3}{8}\left(-\pi\left[\frac{x}{4}\sin 2x\right]_0^{\frac{\pi}{2}} - \pi\left[\frac{x^2}{4} + \frac{1}{8}\cos 2x\right]_0^{\frac{\pi}{2}}\right)$	B1
		$= \left(\frac{\pi^3}{8}\right) - \pi\left(\left[\sin\pi - \sin 0\right] + \left[\frac{\pi^2}{16} + \frac{1}{8}\cos\pi - \frac{1}{8}\cos 0\right]\right)$	DM1
	$= \frac{\pi^3}{16} + \frac{\pi}{4}$		A1
	Complete method to find $\bar{x}$		DM1
	$\bar{x} = \frac{\frac{\pi^3 + 4\pi}{16}}{\frac{\pi^2}{4}} = \frac{\pi^2 + 4}{4\pi}$		A1
			(7)

	(11 marks)
Notes	
(a) M1	Forms the required integral, limits not needed but need to see π or justification at end.
M1	Use of $\sin^2\theta = k(1 \pm \cos 2\theta)$ with $k = \pm\frac{1}{2}$ or ± 2. Limits not needed. π not needed.
DM1	Integration of their function and substitution of correct limits. π may be missing. Integration requires powers of x to increase and sin/cos to become cos/sin. Depends on the first M mark only.
A1*	cso. Penalise incorrect signs on trig functions which become 0 on substitution of the limits. If π is missing but appears only in the last line can only score maximum M0M1M1A0. If explanation for inclusion of π is given all marks are available.
	N.B. the first 5 marks in (b) are available with or without π in front of the integration
(b) M1	Use of $\int y^2 x \, dx$ (limits not needed)
M1	Integration by parts of $x\cos 2x$, or of $x\sin^2 x$ using result from (a), with "u" = kx
B1	Correct integration of polynomial part: $\frac{\pi^3}{16}$ or $\frac{\pi^3}{8}$ (or $\frac{\pi^2}{16}$ or $\frac{\pi^2}{8}$ without π) or equivalent seen or implied.
DM1	Completing the integration. Dependent on the previous M mark.
A1	Correct integration seen or implied N.B. without limits the integral is $\frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x \quad (+c)$
DM1	Use of $\bar{x} = \frac{(\pi)\int y^2 x \, dx}{(\pi)\int y^2 \, dx}$. π must be included for both integrals or neither. Dependent on second M mark i.e. must have attempted integration by parts. Denominator must be the given result from (a) (allow correctly cancelled π) Numerator must use their value.
A1	Cso. Accept any equivalent exact form. N.B. Question specifies need for algebraic working, so e.g. $\pi\int_0^{\frac{\pi}{2}} x\sin^2 x \, dx = \frac{\pi^3}{16} + \frac{\pi}{4}$ followed by answer with no algebraic working would score M1M0B0M0A0M0A0.

6 The path followed by a motorcycle round a circular racetrack is modelled as a horizontal circle of radius 50 m. The track is banked at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The motorcycle travels round the track at constant speed.

The motorcycle is modelled as a particle and air resistance can be ignored.

In an initial model it is assumed that there is no sideways friction between the motorcycle tyres and the track.

Using this model,

(a) find the speed, in m s^{-1}, of the motorcycle.

(5)

In a refined model it is assumed that there is sideways friction.

The coefficient of friction between the motorcycle tyres and the track is $\frac{1}{4}$

It is still assumed that air resistance can be ignored and that the motorcycle is modelled as a particle.

The motorcycle's path is unchanged.

Using this model,

(b) find the maximum speed, in m s^{-1}, at which the motorcycle can travel without slipping sideways.

(8)

(Total for Question 6 is 13 marks)