3. (a) Use algebraic integration to show that the centre of mass of a uniform semicircular lamina of radius r is $\frac{4r}{3\pi}$ from the centre of the semicircle.

[You may assume, without proof, that the area of a circle of radius r is πr^2]

(5)

In the template shown shaded in Figure 2, C_1 is a semicircular lamina with a semicircular lamina C_3 removed. A semicircular lamina C_2 is attached to C_1.

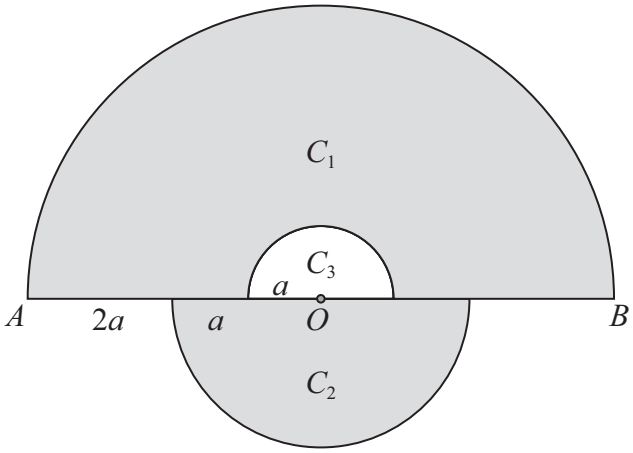


Figure 2

- C_1 has radius $4a$, diameter AB and centre O
- C_2 has radius $2a$ and centre O
- C_3 has radius a and centre O

Laminas C_1 and C_2 are made from the same material and lie in the same plane.

The template is freely suspended from A and hangs in equilibrium with AB at an angle of $\theta°$ to the downward vertical.

(b) Find the value of θ

(7)

(Total for Question 3 is 12 marks)