5.

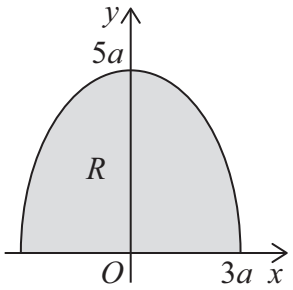


Figure 3

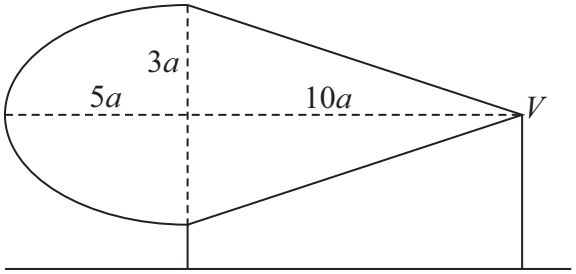


Figure 4

The region R, shown shaded in Figure 3, is the region enclosed between the x-axis and the curve with equation $\frac{x^2}{9a^2} + \frac{y^2}{25a^2} = 1$

Region R is rotated through π radians about the y-axis to form a solid of revolution.
A uniform solid S has the same shape and size as this solid of revolution.

The volume of S is $30\pi a^3$

The distance of the centre of mass of S from O is d.

(a) Use algebraic integration to show that $d = \frac{15}{8}a$.

(6)

The solid S is joined to a uniform solid right circular cone C that is made of the **same material** as S to form a model.
The cone has vertex V, height $10a$ and base radius $3a$.
The plane face of S and the plane face of C coincide.
The model is supported on two light vertical rods.

- the first rod has one end attached to the model at V and the other end attached to horizontal ground
- the second rod has one end attached to the model at a point on the rim of the common plane face and the other end attached to the horizontal ground

The model rests in equilibrium with its axis of symmetry horizontal and in the same vertical plane as the two rods, as shown in Figure 4.

Given that the weight of the model is $2W$

(b) find, in terms of W, the thrust in the rod attached to the model at V.

(4)

(Total for Question 5 is 10 marks)

