6.

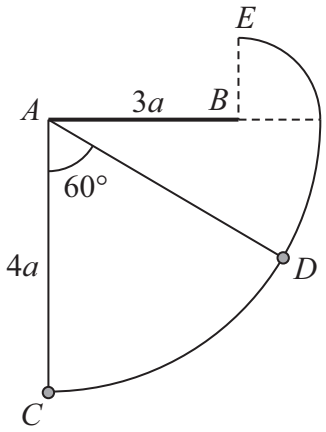


Figure 5

A thin fixed horizontal shelf AB has length $3a$.
A particle of mass m is attached to one end of a light inextensible string of length $4a$.
The other end of the string is attached to the underneath of the shelf at A.
The particle is hanging at rest at C, vertically below A, when it is projected horizontally with speed u.
The particle starts to move, with the string taut, in a vertical plane that is perpendicular to the plane of the shelf. The edge of the shelf is perpendicular to the circular path of the particle.
Air resistance is ignored.
When the string is horizontal, the string wraps round the end of the shelf and the particle continues to move in the vertical plane.
The particle follows the path CDE, as shown in Figure 5.

The point D on the circular path of the particle is such that angle $CAD = 60°$

(a) Find, in terms of m, u, a and g, the tension in the string at the instant when the particle is at D.

(5)

The point E is vertically above B, with $BE = a$.
Given that the particle is still moving in a vertical circle when it reaches E

(b) show that $u \geqslant \sqrt{11ga}$

(3)

At the instant when the particle is at E, the particle breaks free from the string.
The particle moves freely under gravity and first lands on the shelf at the point F.

Given that $u = \sqrt{12ga}$

(c) find, in terms of a, the distance BF.

(4)

(Total for Question 6 is 12 marks)