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5. A continuous random variable X has probability density function

$$f(x) = \begin{cases} k\{(x-2)^2 + 1\} & 0 \leqslant x \leqslant a \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.
Given that the mode of X is 0

(a) find the range of possible values for a.

(2)

Given that $a = 3$

(b) (i) sketch the probability density function for all values of x.

(ii) Give the coordinates of the turning point.

(2)

(c) Use algebraic integration to show that $k = \frac{1}{6}$

(4)

Given that $E(X^2) = 2.1$

(d) use algebraic integration to find $\text{Var}(X)$.

(6)

(e) Show that $P(2 < X < 3) = \frac{2}{9}$

(2)

(f) State, giving a reason, whether the median of X is less than or greater than 1

(3)

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Question 5 continued

(Total 19 marks)

Q5