5. A game uses two turntables, one red and one yellow. Each turntable has a point marked on the circumference that is lined up with an arrow at the start of the game. Jim spins both turntables and measures the distance, in metres, each point is from the arrow, around the circumference in an anticlockwise direction when the turntables stop spinning.

The continuous random variable Y represents the distance, in metres, the point is from the arrow for the yellow turntable. The cumulative distribution function of Y is given by $\mathrm{F}(y)$ where

$$\mathrm{F}(y) = \begin{cases} 0 & y < 0 \\ 1 - (\alpha + \beta y^2) & 0 \leqslant y \leqslant 5 \\ 1 & y > 5 \end{cases}$$

(a) Explain why (i) $\alpha = 1$

(ii) $\beta = -\dfrac{1}{25}$

(2)

(b) Find the probability density function of Y

(2)

The continuous random variable R represents the distance, in metres, the point is from the arrow for the red turntable. The distribution of R is modelled by a continuous uniform distribution over the interval $[d, 3d]$

Given that $\mathrm{P}\left(R > \dfrac{11}{5}\right) = \mathrm{P}\left(Y > \dfrac{5}{3}\right)$

(c) find the value of d

(3)

In the game each turntable is spun 3 times. The distance between the point and the arrow is determined for each spin. To win a prize, at least 5 of the distances the point is from the arrow when a turntable is spun must be less than $\dfrac{11}{5}$ m

Jo plays the game once.

(d) Calculate the probability of Jo winning a prize.

(4)

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