6. The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ ax + bx^2 & 0 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where a, b and k are positive constants.

(a) Show that $ak = 1 - bk^2$

(1)

Using part (a) and given that $\mathrm{E}(X) = \frac{6}{5}$

(b) show that $5bk^3 = 36 - 15k$

(6)

Using part (a) and given that $\mathrm{E}(X) = \frac{6}{5}$ and $\mathrm{Var}(X) = \frac{22}{75}$

(c) show that $5bk^4 = 52 - 10k^2$

(5)

Given that $k < 3$

(d) find the value of k

(4)

(e) Hence find the value of a and the value of b

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

Question 6 continued

(Total for Question 6 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA