2. The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} ax^3 & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant d \\ 0 & \text{otherwise} \end{cases}$$

where a, b, c and d are constants such that

- $bx + c = ax^3$ at $x = 4$
- $bx + c$ is a straight line segment with end coordinates $(4, 64a)$ and $(d, 0)$

(a) State the mode of X

(1)

Given that the mode of X is equal to the median of X

(b) use algebraic integration to show that $a = \frac{1}{128}$

(2)

(c) Find the value of d

(2)

(d) Hence find the value of b and the value of c

(3)

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