7. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} ax & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant 8 \\ 0 & \text{otherwise} \end{cases}$$

where a, b and c are constants.

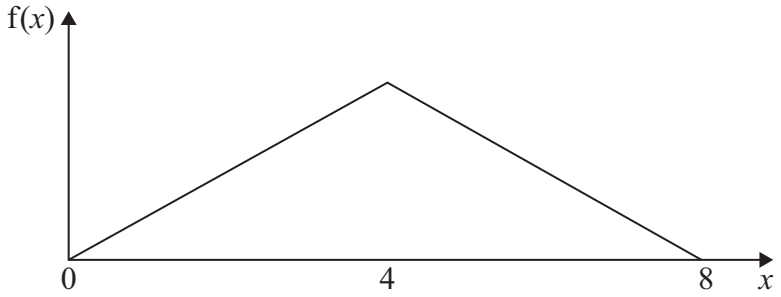


Figure 1

Figure 1 shows the graph of the probability density function f(x)

The graph consists of two straight line segments of equal length joined at the point where $x = 4$

(a) Show that $a = \frac{1}{16}$

(1)

(b) Hence find

(i) the value of b

(ii) the value of c

(3)

(c) Using algebraic integration, show that $\text{Var}(X) = \frac{8}{3}$

(6)

(d) Find, to 2 decimal places, the lower quartile and the upper quartile of X

(3)

A statistician claims that

$$P(-\sigma < X - \mu < \sigma) > 0.5$$

where μ and σ are the mean and standard deviation of X

(e) Show that the statistician's claim is correct.

(2)

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Question 7 continued

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TOTAL FOR PAPER IS 75 MARKS

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