4. (i) The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x^3}{20} & 1 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Using algebraic integration find $E(X^2)$
You must show your working.

(3)

Given that $E(X) = 2.42$

(b) find the value of $\text{Var}(X)$ correct to 3 significant figures.

(2)

(ii) A random sample of size 10 is taken from a continuous random variable Y

(a) Find the probability that at least 7 of these values are **smaller** than the upper quartile of Y

(3)

(b) Find the probability that less than or equal to 5 of these values are **larger** than the upper quartile of Y

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 10 marks)