4. The continuous random variable X has cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{24}(x-1)^2 & 1 \leqslant x \leqslant 3 \\ \frac{1}{6}(x-2) & 3 < x \leqslant 6 \\ -k\left(x^2 - 20x + 52\right) & 6 < x \leqslant 10 \\ 1 & x > 10 \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{1}{48}$

(2)

(b) Write down the value of $P(X = 4)$

(1)

(c) Find the value of

$$P(2.5 < X < 4.5) + P(5.5 < X < 8.5)$$

(3)

The probability density function of X is given by

$$f(x) = \begin{cases} r(x) & 1 \leqslant x \leqslant 3 \\ \frac{1}{6} & 3 < x \leqslant 6 \\ s(x) & 6 < x \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

(d) Find $r(x)$ and $s(x)$

(3)

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Question 4 continued

(Total for Question 4 is 9 marks)