1. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{20}(x^2 - 4) & 2 \leqslant x \leqslant 4 \\ \frac{1}{5}(2x - 5) & 4 < x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

(a) Calculate $P(X > 4)$

(2)

(b) Find the value of a such that $P(3 < X < a) = 0.642$

(4)

(c) Find the probability density function of X, specifying it for all values of x.

(4)

(Total for Question 1 is 10 marks)

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2. Bill owns a restaurant.

Over the next four weeks Bill decides to carry out a sample survey to obtain the customers' opinions.

(a) Suggest a suitable sampling frame for the sample survey.

(1)

(b) Identify the sampling units.

(1)

(c) Give one advantage and one disadvantage of taking a census rather than a sample survey.

(2)

Bill believes that only 30% of customers would like a greater choice on the menu.

He takes a random sample of 50 customers and finds that 20 of them would like a greater choice on the menu.

(d) Test, at the 5% significance level, whether or not the percentage of customers who would like a greater choice on the menu is more than Bill believes.
State your hypotheses clearly.

(5)

(Total for Question 2 is 9 marks)
