4. The function $f(x)$ is defined as

$$f(x) = \begin{cases} \frac{1}{9}(x+5)(3-x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

Albert believes that $f(x)$ is a valid probability density function.

(a) Sketch $f(x)$ and comment on Albert's belief.

(3)

The continuous random variable Y has probability density function given by

$$g(y) = \begin{cases} ky(12 - y^2) & 1 \leqslant y \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(b) Use calculus to find the mode of Y

(3)

(c) Use algebraic integration to find the value of k

(3)

(d) Find the median of Y giving your answer to 3 significant figures.

(3)

(e) Describe the skewness of the distribution of Y giving a reason for your answer.

(2)

(Total for Question 4 is 14 marks)