6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A farmer grows watermelons on a farm. The weights, in kilograms, of the watermelons are modelled by the random variable W with probability density function

$$f(w) = \begin{cases} \frac{1}{9}(6 - w)(w - 2) & 2 \leqslant w \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $f(w)$

(2)

(b) State the mode of W

(1)

(c) Find the mean weight of these watermelons.

(4)

Given that $E(W^2) = \frac{73}{5}$

(d) find the standard deviation of the weights of these watermelons, giving the answer to 3 significant figures.

(2)

(e) Show that the median lies between $w = 3.7$ and $w = 3.8$

(5)

The farmer inspects the watermelons to check whether or not they are spoiled.

The number of spoiled watermelons found by the farmer in an area of $150\,m^2$ on the farm is modelled by a Poisson distribution with mean 5

The farmer randomly selects an area of $900\,m^2$ from the farm and counts the number of spoiled watermelons.

(f) Use a normal approximation to find the probability that the farmer finds fewer than 20 spoiled watermelons.

(5)

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Question 6 continued

(Total for Question 6 is 19 marks)