6. The continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{20}x^3 & 0 \leqslant x \leqslant 2 \\ \frac{1}{10}(6-x) & 2 < x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $\mathrm{f}(x)$ for all values of x

(3)

(b) Write down the mode of X

(1)

(c) Show that $\mathrm{P}(X > 2) = 0.8$

(2)

(d) Define fully the cumulative distribution function $\mathrm{F}(x)$

(4)

Given that $\mathrm{P}(X < a \mid X > 2) = \frac{5}{8}$

(e) find the value of $\mathrm{F}(a)$

(2)

(f) Hence, or otherwise, find the value of a. Give your answer to 3 significant figures.

(3)

(Total for Question 6 is 15 marks)