5. A bag contains a large number of counters with the numbers 4, 6 or 8 written on each of them in the ratio 5 : 3 : 2 respectively.

A random sample of size 3 is taken from the bag.

M represents the median of the numbers on the 3 counters.

(a) List all 7 possible samples of size 3 with $M = 4$

(2)

(b) Find the probability that $M = 4$

(4)

(c) Find the sampling distribution of M

(4)

A random sample of n sets of 3 counters is taken. The random variable Y represents the number of these n sets that have $M = 6$

(d) Calculate the minimum value of n such that $\mathrm{P}(Y \geqslant 1) > 0.95$

(3)

(Total for Question 5 is 13 marks)

---

6. The continuous variable X is uniformly distributed over the interval $[a, b]$ where $0 < a < b$

Given that $\mathrm{E}(X) = \dfrac{5a}{2}$

(a) find $\mathrm{P}(X > b - 2a)$

(4)

The continuous variable Y is uniformly distributed over the interval $[3, c]$ where $c > 3$

Given that $\mathrm{E}\left(Y^2\right) = 549$, find

(b) (i) the value of c

(3)

(ii) the exact value of $\mathrm{P}\left(4Y - 14 < \dfrac{17 - Y}{2}\right)$

(3)

(Total for Question 6 is 10 marks)