5. A random sample of two observations X_1 and X_2 is taken from a population with unknown mean μ and unknown variance σ^2

(a) Explain why $\dfrac{X_1 - \mu}{\sigma}$ is not a statistic.

(1)

(b) Explain what you understand by an unbiased estimator for μ

(1)

Two estimators for μ are U_1 and U_2 where

$$U_1 = 3X_1 - 2X_2 \qquad \text{and} \qquad U_2 = \frac{X_1 + 3X_2}{4}$$

(c) Show that both U_1 and U_2 are unbiased estimators for μ

(3)

The most efficient estimator among a group of unbiased estimators is the one with the smallest variance.

(d) By finding the variance of U_1 and the variance of U_2 state, giving a reason, the most efficient estimator for μ from these two estimators.

(4)

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