1 A machine fills bottles with mineral water.

The machine is checked every day to ensure that it is working correctly. On a particular day a random sample of 100 bottles is taken. The volume of water, x millilitres, for each bottle is measured and each measurement is coded using

$$y = x - 1000$$

The results are summarised below

$$\sum y = 847 \qquad \sum y^2 = 13\ 510.09$$

(a) (i) Show that the value of the unbiased estimate of the mean of x is 1008.47

(ii) Calculate the unbiased estimate of the variance of x

(4)

The machine was initially set so that the volume of water in a bottle had a mean value of 1010 millilitres.

Later, a test at the 5% significance level is used to determine whether or not the mean volume of water in a bottle has changed. If it has changed then the machine is stopped and reset.

(b) Write down suitable null and alternative hypotheses for a 2-tailed test.

(1)

(c) Find the critical region for $\bar{X}$ in the above test.

(4)

(d) Using your answer to part (a) and your critical region found in part (c), comment on whether or not the machine needs to be stopped and reset.
Give a reason for your answer.

(2)

(e) Explain why the use of $\sigma^2 = s^2$ is reasonable in this situation.

(1)

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 12 marks)