3. A random sample of 2 observations, X_1 and X_2, is taken from a population with unknown mean μ and unknown variance σ^2

(a) Explain why $\dfrac{X_1 - X_2}{\sigma}$ is not a statistic.

(1)

$$S = \frac{3}{5}X_1 + \frac{5}{7}X_2$$

(b) Show that S is a biased estimator of μ

(2)

(c) Hence find the bias, in terms of μ, when S is used as an estimator of μ

(1)

Given that $Y = aX_1 + bX_2$ is an unbiased estimator of μ, where a and b are constants,

(d) find an equation, in terms of a and b, that must be satisfied.

(2)

(e) Using your answer to part (d), show that $\text{Var}(Y) = (2a^2 - 2a + 1)\sigma^2$

(3)

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Question 3 continued

(Total for Question 3 is 9 marks)