6: A population has an unknown mean μ and an unknown variance σ^2

A random sample represented by $X_1, X_2, \ldots, X_{20}$ is taken from the population.

(a) Explain why $\frac{1}{2}X_1 + \frac{3}{4}X_{20}$ is a statistic.

(1)

(b) Show that $R = \frac{1}{2}X_1 + \frac{3}{4}X_{20}$ is a biased estimator of μ

(2)

(c) Hence find the bias, in terms of μ, when R is used as an estimator of μ

(1)

Given that $S = \frac{aX_1 - bX_2}{2}$ and $T = \frac{\sum_{i=1}^{14} aX_i + \sum_{i=15}^{20} bX_i}{20}$ are each used as an unbiased estimator of μ, where a and b are constants,

(d) find the value of a and the value of b

(4)

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Question 6 continued

(Total for Question 6 is 8 marks)