| Question Number | Scheme | Marks |
|---|---|---|
| 3(a) | It involves an **unknown** (population) **parameter** $\sigma$ | B1 |
| | | **(1)** |
| (b) | $\left[\mathrm{E}(S)=\mathrm{E}\left(\frac{2}{5}X_1+X_2-\frac{1}{3}X_3\right)\right]=\frac{2}{5}\mathrm{E}(X_1)+\mathrm{E}(X_2)-\frac{1}{3}\mathrm{E}(X_3)=\frac{2}{5}\mu+\mu-\frac{1}{3}\mu$ | M1 |
| | $\frac{2}{5}\mu+\mu-\frac{1}{3}\mu=\frac{16}{15}\mu\neq\mu$ (so $S$ is a biased estimator of $\mu$) | A1 |
| | | **(2)** |
| (c) | Bias $E(S)-\mu=\text{"}\frac{16}{15}\mu\text{"}-\mu=\frac{1}{15}\mu$ | B1ft |
| | | **(1)** |
| (d) | $\mathrm{E}(Y)=a\mathrm{E}(X_1)+b\mathrm{E}(X_2)-3\mathrm{E}(X_3)=\mu$<br>$\Rightarrow(a+b-3)\mu=\mu$ | M1 |
| | $a+b=4$ | A1 |
| | | **(2)** |
| (e) | $\mathrm{Var}(Y)=a^2\mathrm{Var}(X_1)+b^2\mathrm{Var}(X_2)+(-3)^2\mathrm{Var}(X_3)=\left(a^2+b^2+9\right)\sigma^2$ | M1 |
| | $\mathrm{Var}(Y)=\left(a^2+(4-a)^2+9\right)\sigma^2$ | M1 |
| | $\mathrm{Var}(Y)=\left(2a^2-8a+25\right)\sigma^2$ | A1 |
| | | **(3)** |
| (f) | $\mathrm{Var}(Y)=\left(2(a-2)^2+17\right)\sigma^2$ | M1 |
| | Smallest value of $\mathrm{Var}(Y)=17\sigma^2$ | A1ft |
| | | **(2)** |

| | Notes | | Total 11 |
|---|---|---|---|
| (a) | **B1** | Suitable comment referring to the bold words and including $\sigma$ Ignore any references to $\mu$ | |
| (b) | **M1** | Writing or using $\mathrm{E}(S)=\frac{2}{5}\mathrm{E}(X_1)+\mathrm{E}(X_2)-\frac{1}{3}\mathrm{E}(X_3)$. Must be in terms of E(X) or $\mu$.<br>Implied by correct expression $\frac{2}{5}\mu+\mu-\frac{1}{3}\mu$ (o.e). Condone missing subscripts. | |
| | **A1** | cao<br>Must see $\frac{16}{15}\mu$ or awrt $1.07\mu$ and showing that is does not equal $\mu$ (e.g. $\frac{16}{15}\mu\neq\mu$) | |
| (c) | **B1ft** | $\frac{1}{15}\mu$ (awrt $0.07\mu$) or follow through their part (b) $-\mu$ | |
| (d) | **M1** | For writing or using $\mathrm{E}(Y)=a\mathrm{E}(X_1)+b\mathrm{E}(X_2)-3\mathrm{E}(X_3)=\mu$. Implied by $a+b=4$.<br>Condone missing subscripts. | |
| | **A1** | cao | |
| | **SC** | If they state, $a=b=2$ then score **M1A1** | |
| (e) | **M1** | Writing or using $\mathrm{Var}(Y)=a^2\mathrm{Var}(X_1)+b^2\mathrm{Var}(X_2)+(-3)^2\mathrm{Var}(X_3)$. Condone missing subscripts. | |
| | **M1** | For substituting of $b=4-a$ ft their part (d) into their expression for $\mathrm{Var}(Y)$ | |
| | **A1** | $\mathrm{Var}(Y)=\left(2a^2-8a+25\right)\sigma^2$ (o.e) | |
| | **SC** | Using -9 $\mathrm{Var}(X_3)$ instead of (-3)$^2$ $\mathrm{Var}(X_3)$ leads to $\left(2a^2-8a+7\right)\sigma^2$ **M1M1A0** | |

| | | |
|---|---|---|
| **(f)** | **M1** | For a valid method to find minimum e.g. differentiation, complete the square.<br>Implied by 17 seen if their values of $p$, $q$ and $r$ are correct. |
| | **A1ft** | $17\sigma^2$ or ft their $a$ and using the correct expression $\left(2a^2-8a+25\right)\sigma^2$. $\text{Var}(Y)<0$ A0 |