3. A random sample of 3 observations, X_1, X_2 and X_3 are taken from a population with unknown mean $\mu (\mu \neq 0)$ and unknown variance σ^2

(a) Explain why $\dfrac{X_1 + X_2 - X_3}{\sigma}$ is not a statistic.

(1)

Given that

$$S = \frac{2}{5}X_1 + X_2 - \frac{1}{3}X_3$$

(b) show that S is a biased estimator of μ

(2)

(c) Hence find the bias, in terms of μ, when S is used as an estimator of μ

(1)

Given that $Y = aX_1 + bX_2 - 3X_3$ is an unbiased estimator of μ, where a and b are constants,

(d) find, in terms of a and b, an equation that must be satisfied.

(2)

Using the answer to part (d),

(e) show that

$$\text{Var}(Y) = (pa^2 + qa + r)\,\sigma^2$$

where p, q and r are constants to be found.

(3)

(f) Hence, or otherwise, find the smallest possible value of $\text{Var}(Y)$, giving your answer in terms of σ^2

(2)

(Total for Question 3 is 11 marks)