Question	Answer	Marks	Guidance
8(b)	**Alternative 2** Use of $\text{cosec}^2\theta = 1 + \cot^2\theta$ **and** $\cot^2\theta = \frac{1}{\tan^2\theta}$	**(B1)**	May be implied by later work
	$\tan^2\theta = \frac{4-\sqrt{3}}{7+4\sqrt{3}}$ $= 52 - 30\sqrt{3}$	**(2)**	**M1** for attempting to rationalise $\tan^2\theta$ with sufficient detail in the simplification of the numerator – at least 3 terms
	$\text{cosec}^2\theta = 1 + \left(\frac{1}{52-30\sqrt{3}}\right)^2$ $= 14 + \frac{15\sqrt{3}}{2}$	**(2)**	**M1** for attempting to rationalise $\cot^2\theta$ with sufficient detail in the simplification of the numerator – at least 3 terms **and** expressing as a single fraction and attempt to simplify to required form.
	Alternative 3 Use of right-angled triangle $\text{Hyp}^2 = 11 + 2\sqrt{3}$	**(2)**	**M1** For attempt to calculate the square of hypotenuse
	$\text{cosec}^2\theta = \frac{11+2\sqrt{3}}{4-2\sqrt{3}}$	**(B1)**	for correct use of $\text{cosec}^2\theta$ with *their* squared hypotenuse
	$\text{cosec}^2\theta = \frac{11+2\sqrt{3}}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$	**(M1)**	For attempt to rationalise *their* expression, with sufficient detail in the simplification of the numerator – at least 3 terms
	$14 + \frac{15\sqrt{3}}{2}$	**A1**	
9(a)	$\frac{1}{2}r^2\theta = 10,\ \theta = \frac{20}{r^2}$	**B1**	
	$[P =]\ 2r + r\theta$	**M1**	For substituting *their* θ in P
	$[P =]\ 2r + \frac{20}{r^2}$	**A1**	
9(b)	$\frac{dP}{dr} = 2 - \frac{20}{r^2}$	**M1**	For attempt to differentiate *their* answer to part **(a)** to obtain the form of $\left[\frac{dP}{dr} =\right] 2 + \frac{a}{r^2}$
	When $\frac{dP}{dr} = 0,\ r = \sqrt{10}$	**2**	**Dep M1** for equating *their* $\frac{dP}{dr}$ to zero and attempt to solve **A1** cao

Question	Answer	Marks	Guidance
9(c)	$\frac{d^2P}{dr^2} = \frac{40}{r^3}$ As r is positive, $\frac{d^2P}{dr^2}$ is also positive so minimum	**2**	**M1** for a complete method, allow valid alternatives, if differentiated, must be in the form of $\left[\frac{d^2P}{dr^2} = \right]\frac{k}{r^3}$ **A1** for a correct conclusion
9(d)	$\theta = 2$	**B1**	
10	$-1 = \tan\left(3p + \frac{\pi}{2}\right)$ $p = \frac{\pi}{12}$	**2**	**M1** for a complete method to find the value of p
	$\frac{dy}{dx} = 3\sec^2\left(3x + \frac{\pi}{2}\right)$	**2**	**M1** for $a\sec^2\left(3x + \frac{\pi}{2}\right)$ **A1** all correct
	When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 6$	**M1**	For attempt to find the gradient using *their p* from differentiation
	Equation of normal: $y + 1 = -\frac{1}{6}\left(x - \frac{\pi}{12}\right)$	**M1**	For attempt at normal equation using *their p* and $-\frac{1}{\textit{their}\text{ value for }\frac{dy}{dx}}$
	When $x = 0$, $y = \frac{\pi}{72} - 1$	**M1**	For attempt to find B using *their* normal equation (must be from differentiation)
	When $y = 0$, $x = \frac{\pi}{12} - 6$	**M1**	For attempt to find A using *their* normal equation (must be from differentiation)
	Mid-point $\left(\frac{\pi}{24} - 3,\ \frac{\pi}{144} - \frac{1}{2}\right)$	**2**	**A1** for x value (must be exact) **A1** for y value (must be exact)

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