| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $9kx+1=kx^2+3(2k+1)x+4$, leading to $kx^2+x(3-3k)+3 \quad [=0]$ | **M1** | For equating the two equations and attempt to obtain a 3 term quadratic equation equated to zero. |
| | $(3-3k)^2-(4\times 3k)$ oe | **M1** | **Dep** on previous M mark for attempt to use the discriminant in any form |
| | $3k^2-10k+3$ oe | **M1** | **Dep** on previous M mark for simplification to a 3 term quadratic expression in terms of $k$ |
| | Critical values 3 and $\frac{1}{3}$ | **A1** | For both |
| | $\frac{1}{3}<k<3$ | **A1** | Mark the final answer |
| 2 | $x=\frac{-(2\sqrt{3}+5)\pm\sqrt{(2\sqrt{3}+5)^2-4(3-5\sqrt{3})(-1)}}{2(3-5\sqrt{3})}$ | **M1** | For the use of the quadratic formula |
| | $x=\frac{-(2\sqrt{3}+5)\pm\sqrt{12+20\sqrt{3}+25+12-20\sqrt{3}}}{2(3-5\sqrt{3})}$ | **M1** | For expansion of the square root, must see at least 4 terms |
| | $x=\frac{-12-2\sqrt{3}}{2(3-5\sqrt{3})}$ oe, $x=\frac{2-2\sqrt{3}}{2(3-5\sqrt{3})}$ oe | **A1** | For both |
| | $x=\frac{-12-2\sqrt{3}}{2(3-5\sqrt{3})}\times\frac{3+5\sqrt{3}}{3+5\sqrt{3}}$ oe<br>or $x=\frac{2-2\sqrt{3}}{2(3-5\sqrt{3})}\times\frac{3+5\sqrt{3}}{3+5\sqrt{3}}$ oe<br>with an attempt to simplify | **M1** | For attempt to rationalise at least one of *their* solutions (must be similar structure)<br>Sufficient detail must be seen, at least 3 terms in the numerator |
| | $\frac{1}{2}+\frac{\sqrt{3}}{2}$ | **A1** | Must have sufficient detail shown |
| | $\frac{2}{11}-\frac{\sqrt{3}}{33}$ | **A1** | Must have sufficient detail shown |

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| 3(a) | $b = \frac{1}{8}$ | **B1** | |
| | $11 = a\sin\frac{4\pi}{8} + c$<br>$5 = a\sin\left(\frac{-4\pi}{3\times 8}\right) + c$ | **M1** | For attempt to form two simultaneous equations using given points, together with an attempt to obtain at least one unknown. Allow use of *their b*. |
| | $a = 4$ | **A1** | |
| | $c = 7$ | **A1** | |
| 3(b) | Using symmetry | **M1** | For e.g. period is $16\pi$, symmetrical about the line $x = 8\pi$ |
| | For obtaining max at $4\pi$ and min at $12\pi$ | **M1** | |
| | $x = 12\pi$ | **A1** | |
| | $y = 3$ | **A1** | |
| | **Alternative method 1** | | |
| | Minimum value when $y = 3$ | **(B2)** | **FT** on *their* $-a + c$ |
| | When $y = 3$, $x = 12\pi$. | **(2)** | **M1** for attempt to solve *their* $3 = a\sin bx + c$ using *their* values of *a*, *b* and *c* to get *x* = ... |
| | **Alternative method 2** | | |
| | Min occurs $\frac{3}{4}$ through sine cycle so $x = 12\pi$ | **(B2)** | |
| | When $x = 12\pi$, $y = 3$ | **(2)** | **M1** for attempt to solve $y = a\sin b(12\pi) + c$ using *their* values of *a*, *b* and *c* |
| | **Alternative method 3** | | |
| | $\frac{dy}{dx} = ab\cos bx$<br>$(ab)\cos bx = 0$ | **(M1)** | |
| | $x = 4\pi,\ 12\pi$ | **(M1)** | **Dep** for attempt to solve to obtain *x* = |
| | $x = 12\pi$ | **(A1)** | |
| | $y = 3$ | **(A1)** | cao |