4 The polynomial $\mathrm{p}(x)$ is such that $\mathrm{p}(x) = ax^3 + 13x^2 + bx + c$, where a, b and c are integers. It is given that $\mathrm{p}'(0) = -9$.

(a) Show that $b = -9$. [1]

It is also given that $3x + 2$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $x + 1$ the remainder is 6.

(b) Find the values of a and c. [4]

(c) Find the quadratic $\mathrm{q}(x)$ such that $\mathrm{p}(x) = (3x + 2) \times \mathrm{q}(x)$. [1]

(d) Hence find $\mathrm{p}(x)$ as a product of linear factors with integer coefficients. [1]

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5 A geometric progression is such that the fifteenth term is equal to $\frac{1}{8}$ of the twelfth term. The sum to infinity is 5.

(a) Find the first term and the common ratio. [4]

(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999. [3]

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